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## METHODS AND MODELS OF SURFACE FORMATION IN THE PROCESS OF THREE-DIMENSIONAL MODELING

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### ABSTRACT

*This article analyzes surface formation models in three-dimensional modeling. The processes of three-dimensional modeling are based on several methods and algorithms. Unique models are also used to model various processes and an object which is the capabilities of three-dimensional modeling technology is used in almost all areas of society.*

**KEYWORDS:** 3D Graphics, Surface, Spline, Analytical Model, Polygonal Model, Voxel Model, Bezier Spline, Linear Segments, Array, Vector, Polyline, Polygons, Polygonal Surfaces, Vertex, Corner, Memory Consumption, Computed Tomography, Computer Graphics Systems, Three-Dimensional Cartesian Coordinate System, Angle Indices.

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### INTRODUCTION

Nowadays, in order to better understand the concept of three-dimensional graphics, it is necessary to analyze the data related to this concept. In some publications, this concept is also called 3D graphics. However, it should be noted that it is not entirely correct to call this concept by such a name, because here we are talking about the reflection of the image in the plane, not in size. Real three-dimensional imaging methods are not yet widespread enough.

Three-dimensional graphics is widely used in areas such as scientific researches, engineering design, and computer modeling of objects. As well as, the field of three-dimensional modeling is expanding day by day.

Let's look at how to show the shape of three-dimensional objects in computer graphics systems. Different methods can be used to describe the shape of surfaces. In this paper, we will look at some of the methods and algorithms of three-dimensional modeling.

#### Model for Describing Surfaces. Analytical Model

Analytical model means to describe a surface with mathematical formulas. Many variants of this description can be used in computer graphics. For example,  $z = f(x, y)$  can be taken as a function of two arguments. The equation  $F(x, y, z) = 0$  can be used.

Most often, the parametric form of the surface description is used. We write formulas for the system of three-dimensional decart coordinates  $(x, y, z)$ :

$$x = F_x(s, t),$$

$$y = F_y(s, t),$$

$$z = F_z(s, t),$$

Here,  $s$  and  $t$  determine the surface formula of variable parameters and  $F_x$ ,  $F_y$  and  $F_z$  functions in a certain range.

The advantage of the parametric description is that it is easy to describe surfaces that are suitable

for indefinite functions, closed surfaces. The definition can be made so that when the surface is turned or massaged, the formula does not change significantly.

As an example, we consider an analytical description of the surface of the sphere.

To begin with, we describe as a function of two arguments the following:

$$Z = \pm \sqrt{R^2 - x^2 - y^2}.$$

After that, we define the equation as follows:

$$x^2 + y^2 + z^2 = R^2 = 0.$$

Also in parametric form:

$$x = R \sin s \cos t,$$

$$y = R \sin s \sin t,$$

$$z = R \cos s.$$

Splines are often used to describe complex surfaces. Splayn is a special function that is most suitable for zooming individual parts of the surface. Several splines form a complex surface model. In other words, splayn is also a surface, but the coordinates of its points can be easily calculated. Usually cubic splines are applied. Why exactly Cube? The reason is that the third level is the smallest of the levels, which allows you to depict any shape, and when the splines are combined, you can provide a continuous first formation - such a surface will be without folds on the joints. Splayns are often parametrically defined. We write the formula of the component x (s, t) of the cube splayn in the form of a third-degree polygon of the parameters s and t:

$$\begin{aligned} x(s, t) = & a_{11} s^3 t^3 + a_{12} s^3 t^2 + a_{13} s^3 t + a_{14} s^3 + \\ & + a_{21} s^2 t^3 + a_{22} s^2 t^2 + a_{23} s^2 t + a_{24} s^2 + \\ & + a_{31} s t^3 + a_{32} s t^2 + a_{33} s t + a_{34} s + \\ & + a_{41} t^3 + a_{42} t^2 + a_{43} t + a_{44}. \end{aligned}$$

In the mathematical literature, you can learn how to determine the  $a_{ij}$  coefficients for splines that give properties. Examples of matrix analysis and synthesis are given in.

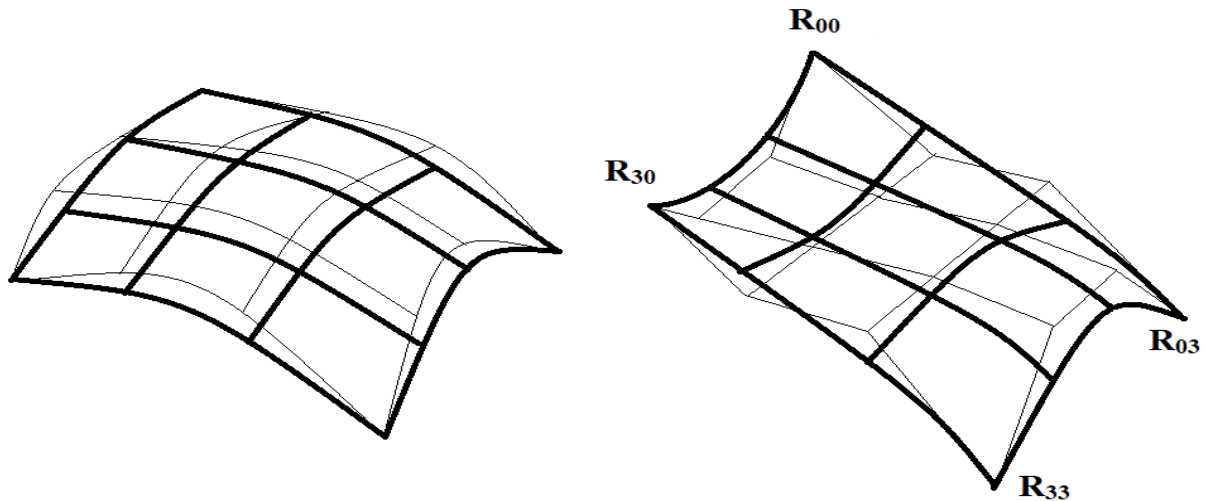
Below is one of the types of spline - Bezier spline. Initially, a generalized form of this spline - the level of  $m * n$  is presented:

$$R(s, t) = \sum_{i=0}^m C_m^i S^i (1 - S)^{m-i} \sum_{j=0}^n C_n^j t^j (1 - t)^{n-j} P_{ij}$$

Here  $R_{ij}$  - turning points - landmarks,  $0 \leq s \leq 1$ ,  $0 \leq t \leq 1$ ,  $C_m^i$  and  $C_n^j$  - Newton binomial coefficients, which are calculated according to the following formula:

$$C_a^b = \frac{a!}{b!(a-b)!}$$

The cubic bezier spline corresponds to the values of  $m = 3$ ,  $n = 3$ . It takes 16 points to identify it -  $R_{ij}$  landmarks (Figure 1); The coefficients  $C_m^i$  and  $C_n^j$  are equal to 1, 3, 3, 1 for  $i, j = 0, 1, 2, 3$ .



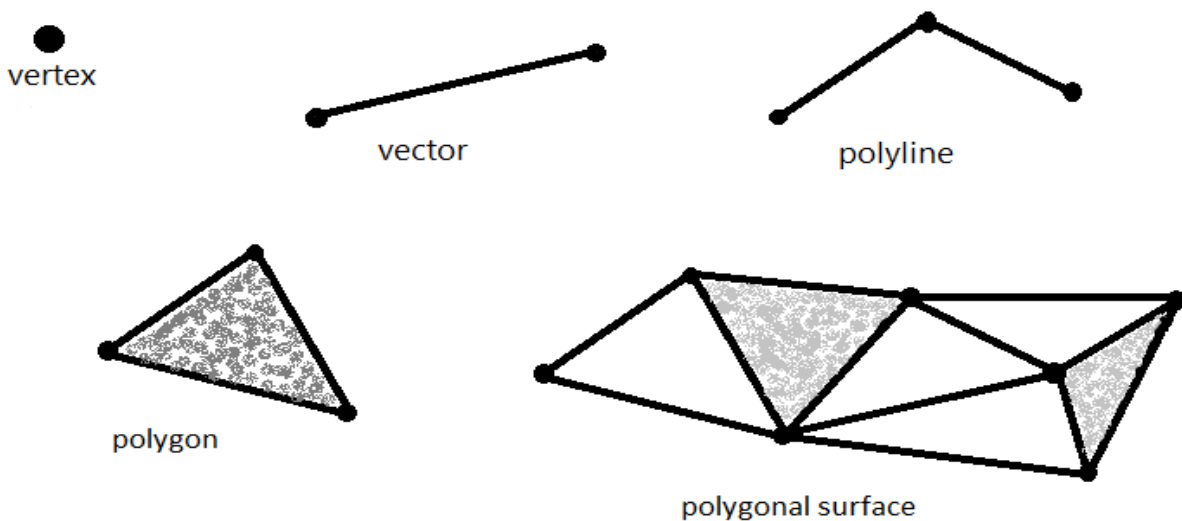
Picture.1. Cube bezier spline

Describing the analytical model as a whole, we can say that this model is one of the optimal solutions suitable for many surface analysis operations. From the point of view of computer graphics, the following positive features of the model can be shown: an easy procedure for calculating the coordinates of each point on the surface, the normal; a small amount of data to describe very complex shapes.

Disadvantages include: complex descriptive formulas using slow computing functions on the computer, reducing the speed of display operations; in many cases it is not possible to use this descriptive form to create a direct surface image. In such cases, the surface is displayed as a polygon, with analytical description formulas used to calculate the coordinates of the surface ends during the mapping process, which reduces the speed relative to the polygonal description model.

**Vector Polygonal Model**

Typically, the following elements are used to describe spatial objects: points; linear segments (vectors); polyline, polygons (polygons); polygonal surfaces (polygonal surfaces) (Fig. 2).



Picture.2. Basic elements of the vector-polygonal model

The dot element is the main element of the description, and the remaining elements are formed by

memorizing exactly three elements. When using a three-dimensional Cartesian system, it is defined as three coordinates (xi, yi, zi). Each object is uniquely defined by the coordinates of its peaks.

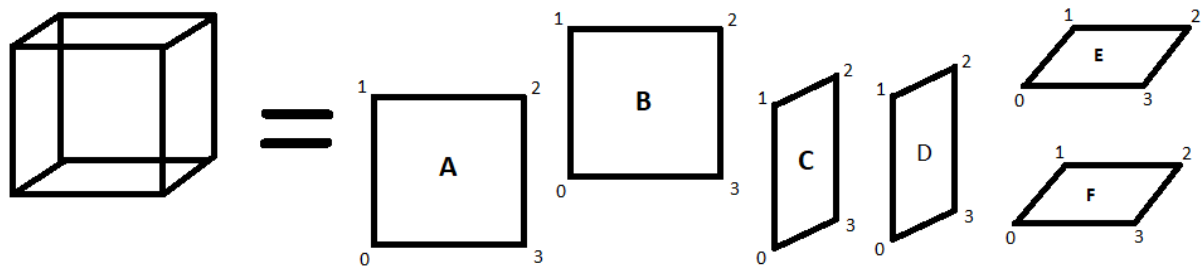
The point element can model a single point property that is not important, and can also be used as an endpoint for a linear object and polygons. The two points define the vector. Several vectors form a polyline. It can model a single-line object that does not take into account the thickness of the polyline and also represent the contour of the polygon. The polygon models the field object. A single polygon can represent the flat surface of a three-dimensional object. Several surfaces form a three-dimensional object in the form of a polygonal surface, a polygonal or open surface. This element is sometimes referred to as a “polygon net”.

The vector polygonal model can be considered the most common in modern 3D computer graphics systems. It is used in computer-aided design systems, computer games and simulators, geographic information systems, and more.

**Vector Polygon Model The First Method**

If the data structures used in the vector polygonal model are considered, these can be done in several ways. For example, a cube is taken as an object, and how the description of such an object can be organized in data structures is considered.

The first method. All surfaces are stored separately (Figure 3).



**Picture. 3. The first way to describe a cube**

$$\text{EdgeA} = \{(x_{A0}, y_{A0}, z_{A0}), (x_{A1}, y_{A1}, z_{A1}), (x_{A2}, y_{A2}, z_{A2}), (x_{A3}, y_{A3}, z_{A3})\}$$

$$\text{EdgeB} = \{(x_{B0}, y_{B0}, z_{B0}), (x_{B1}, y_{B1}, z_{B1}), (x_{B2}, y_{B2}, z_{B2}), (x_{B3}, y_{B3}, z_{B3})\}$$

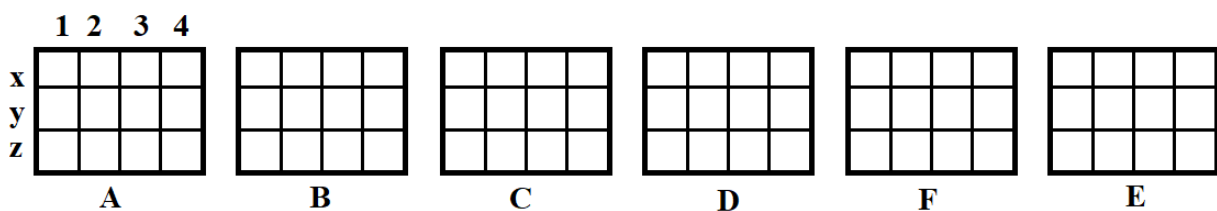
$$\text{EdgeC} = \{(x_{C0}, y_{C0}, z_{C0}), (x_{C1}, y_{C1}, z_{C1}), (x_{C2}, y_{C2}, z_{C2}), (x_{C3}, y_{C3}, z_{C3})\}$$

$$\text{EdgeD} = \{(x_{D0}, y_{D0}, z_{D0}), (x_{D1}, y_{D1}, z_{D1}), (x_{D2}, y_{D2}, z_{D2}), (x_{D3}, y_{D3}, z_{D3})\}$$

$$\text{EdgeE} = \{(x_{E0}, y_{E0}, z_{E0}), (x_{E1}, y_{E1}, z_{E1}), (x_{E2}, y_{E2}, z_{E2}), (x_{E3}, y_{E3}, z_{E3})\}$$

$$\text{EdgeF} = \{(x_{F0}, y_{F0}, z_{F0}), (x_{F1}, y_{F1}, z_{F1}), (x_{F2}, y_{F2}, z_{F2}), (x_{F3}, y_{F3}, z_{F3})\}$$

We illustrate this schematically in Figure 4.



**Picture 4 Separately obtained surfaces**

This method of describing an object in a computer program can be done differently. For example,

a separate array opens in the computer’s memory for each surface. You can write all surfaces in one array - a vector.

Classes can be used to describe individual surfaces and objects in general. In this case, you can create structures that combine the coordinates (x, y, z) or save the coordinates separately. In many ways, it depends on the skill of the programmer. In fact, in one way or another, it is not much different, only the coordinates of the ends of the surfaces and some information have to be overloaded in memory.

The amount of memory required to represent a cube is calculated as follows:

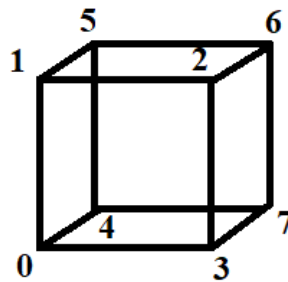
$$P_1 = 6 \times 4 \times 3 \times R_v,$$

Where  $R_v$  is the number of digits required to represent the coordinates.

Here, six surfaces are depicted with 24 angles (Fig. 4). Such an image is a plus - each corner is written three times. This does not take into account the fact that the surfaces have common ends.

**Vector polygon model. The second method**

The second method of description. For this method, the coordinates of the octagon are stored without repetition. The angles are numbered (Figure 5) and each surface is given as a list of angle indices (pointers to the angles) (Figure 6).



Picture 5. Corner numbers

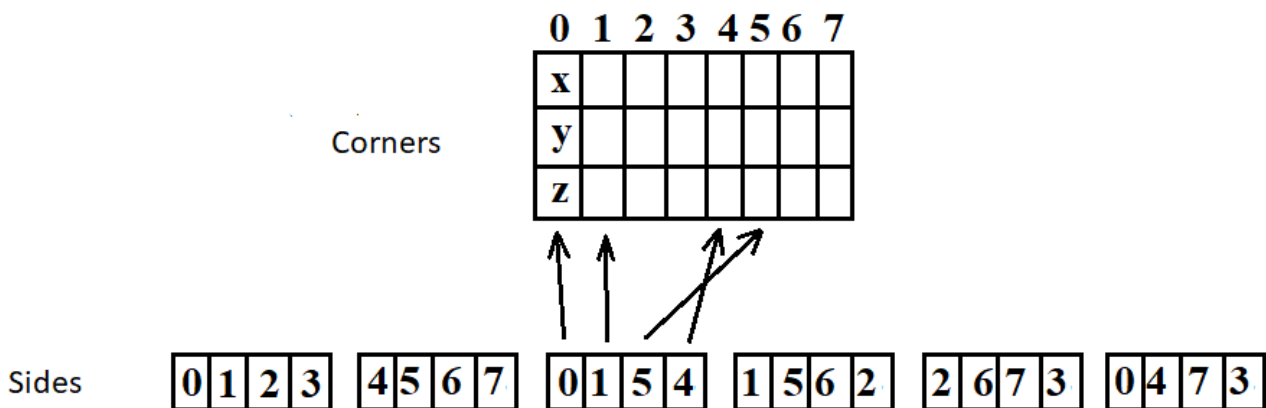


Figure 6. Angle indices are stored in surface arrays

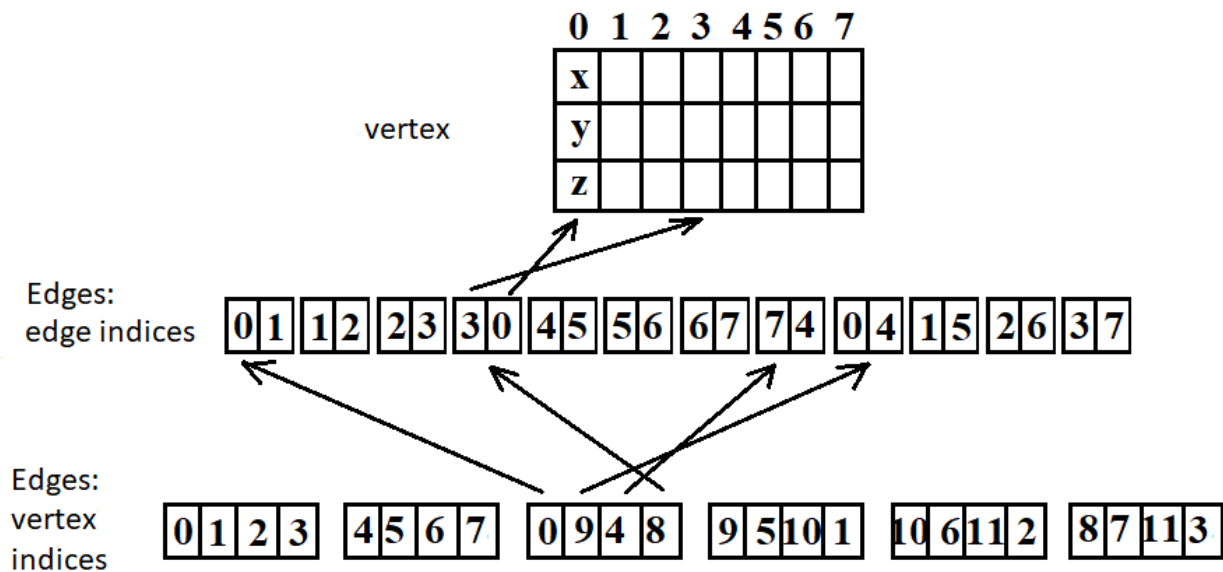
Memory consumption is:

$$P_2 = 8 \times 3 \times R_v + 6 \times 4 \times R_{\text{indeks}},$$

Here  $R_v$  is the bit depth of the angular coordinates,  $R_{\text{indeks}}$  is the bit depth of the indices.

**Vector polygon model. The third method.**

The third method of description (Figure 7).



Picture 1. Linear node model

This method (sometimes called the linear-node model) is based on a hierarchy: angles - edges - surfaces.

Memory consumption is:

$$P_3 = 8 \times 3 \times R_v + 12 \times 2 \times R_{\text{vertex indices}} + 6 \times 4 \times R_{\text{edge indices}},$$

Where  $R_v$  is the bit depth of the angular coordinates,  $R_{\text{vertex indices}}$  and  $R_{\text{edge indices}}$  are the bit depth of the indexes and index of connections, respectively.

To compare the memory size of these three methods, it is necessary to determine the bit depth of the data. Suppose the bit width of the coordinates and indices is four bytes. This corresponds, for example, to a float variable point type for coordinates and a long integer type for indexes. Then the memory consumption in bytes is calculated:

$$P_1 = 6 \times 4 \times 3 \times 4 = 288,$$

$$P_2 = 8 \times 3 \times 4 + 6 \times 4 \times 4 = 192,$$

$$P_3 = 8 \times 3 \times 4 + 12 \times 2 \times 4 + 6 \times 4 \times 4 = 288.$$

8 bytes are allocated for coordinates and 4 bytes for indexes. Then:

$$P_1 = 6 \times 4 \times 3 \times 8 = 576,$$

$$P_2 = 8 \times 3 \times 8 + 6 \times 4 \times 4 = 288,$$

$$P_3 = 8 \times 3 \times 8 + 12 \times 2 \times 4 + 6 \times 4 \times 4 = 384.$$

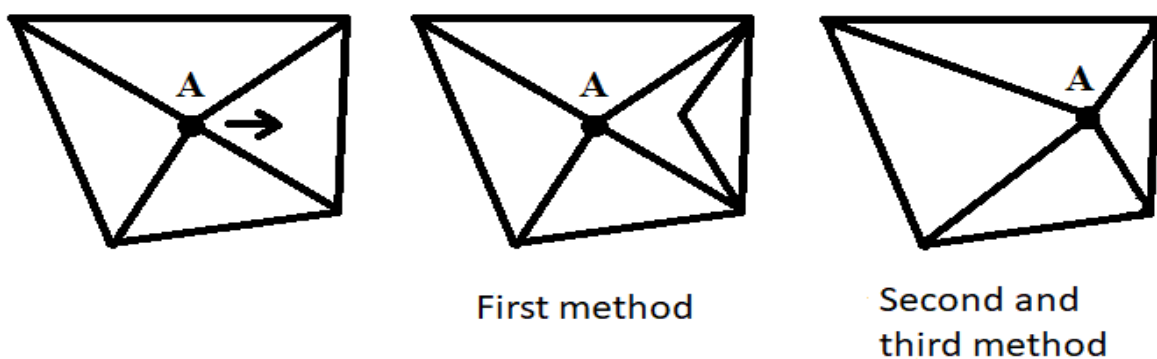
If the bit depth is greater for coordinates than for indexes, the advantage of the second and third methods is significant. Also, the second method can be considered the most economical. It should be noted that this conclusion was made exactly for the cube. The method ratio may be different for other types of objects. In addition, the following methods of constructing data structures should be considered: whether a single array is used for all objects or a separate array is designed for each object (for example, with an object-oriented programming method, each object can be stored in a separate class). This can lead to different desired bit depths for indexes.

### Vector polygonal Model

In conclusion, we compare the vector polygonal model discussed in the previous steps, taking into account three different aspects.

Landfill output speed. If you need to draw a contour line and fill points for polygons, then the first and second methods are close in speed - both the contours and the filler are drawn in the same way. The difference is that for the second method, you must first select a point index that slows down the output process. In both cases, the general part of the contour is redrawn for adjacent surfaces. A more perfect method of contour drawing can be provided for the third method - if bits are given in the connection description arrays, each line is drawn only once, i.e. this connection is assumed to have already been drawn. This defines the advantages of the third method in terms of speed.

Topological aspect. Figure 7 shows what happens if there are multiple adjacent surfaces and the coordinates of a single angle in the data structures are changed.



**Picture 7. the case when the coordinates of a single angle are changed.**

For the second and third methods, since each angle is stored in one copy, changing its coordinates automatically changes all surfaces, with the index of that angle stored in the description. It is used, for example, in geographic information systems to describe adjacent plots of land or other adjacent objects.

It should be noted that a similar result can be achieved with a data structure corresponding to the first method. You can provide a search for other angles whose coordinates correspond to the coordinates of point A.

In other words, support for such an operation can be provided both by data structures and algorithmically.

However, when adjacent surfaces need to be separated, this process is more difficult than the first for the second and third methods, as arrays need to write new angles, new edges, and determine indices in surface arrays.

#### **Advantages of the vector polygonal model:**

- Ease of scaling objects. When zoomed in or out, objects look better than raster description models. The scale range is determined by the number of digits to represent the accuracy of the approximation and the coordinates of the ends;
- Small amounts of data to describe simple surfaces that are approximated in accordance with flat surfaces;
- The need to calculate only the coordinates of the angles when changing coordinate systems or moving objects;



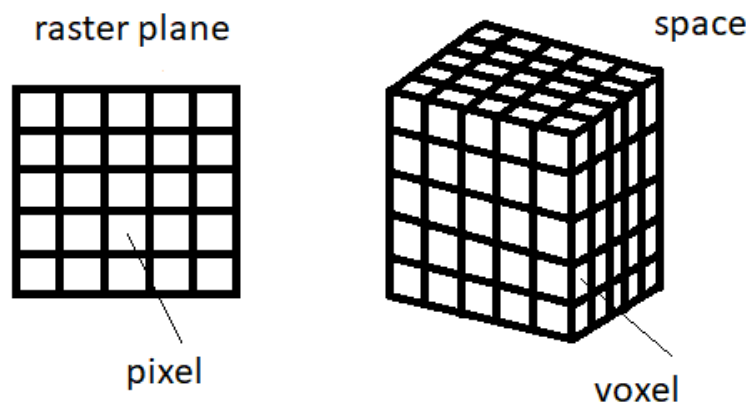
- Hardware for many operations on modern graphics video systems that provide sufficient speed for animation.

**Disadvantages of the vector polygonal model:**

- Complex visualization algorithms for creating real images;
- Complex algorithms for performing topological operations such as intersections;
- Approaching with flat surfaces leads to modeling errors. When modeling surfaces with complex fractal shapes, it is usually not possible to increase the number of surfaces due to computer speed and memory limitations.

**Voxel Model**

The Voxel model is a three-dimensional raster. Just as pixels are located in the 2D image plane, voxels also form three-dimensional objects of a certain size (Figure 8).



**Picture.8. Pixels and voxels**

The voxel is a volume element. It is known that each pixel must have its own color. Each voxel also has its own color and additionally transparency. A voxel with full transparency means that the point of the volume corresponding to that voxel is empty. In volume modeling, each voxel represents a volume element of a certain size. The more voxels of a certain size and the smaller the size of the voxels, the more accurately the 3D objects are modeled.

The voxel method is considered to be one of the most promising methods for modern computer graphics. This method is commonly used in computer systems for medicine. For example, when scanning with a tomograph (computed tomography), images of sections of an object are taken, which are then converted into a three-dimensional model for further analysis. In addition, the voxel method is used in geology, seismology, and computer games. Voxels are also used for graphic display devices that create real three-dimensional images.

**Advantages of the Voxel Model:**

- 3D scenes have a simple process of displaying complex objects and scenes;
- Simple performance of topological operations on individual objects and the whole scene. For example, the section is displayed simply - it is possible to make the corresponding voxels transparent.

**Disadvantages of the Voxel Model:**

- A large amount of data will be required to display the volume data. For example, to depict an object measuring 256 x 256 x 256, more than 16 million voxels have to be used;



- Significant memory costs limit the resolution and accuracy of modeling; a large number of voxels results in a low speed of creating images of three-dimensional scenes;
- As with any raster, there are problems zooming in or out on the image. For example, the image size deteriorates when magnified.

## CONCLUSION

In the current article, we have considered methods for creating surfaces in three-dimensional modeling. In reviewing these scientific analyzes, the specificity of each model and method was studied, analyzed, and the methods of formation were compared. The mathematical basis of surface formation models in three-dimensional modeling was considered. The advantages and disadvantages of the methods were summarized. That is, each method is used to develop the industry. For example, the voxel model and its algorithms are suitable for creating structurally complex objects. A complex object is the process of modeling a person's internal organs in the process of medical research.

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